Game Theory: Activities Motivate Concepts

MathFest Workshop

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Presenters

David is a professor of mathematics and computer science at Goshen College in northern Indiana. His Ph.D. is in game theory. He has taught modeling and game theory courses throughout his career.

Rick is a retired professor of mathematics from Valparaiso University (in northwest Indiana) and he did not become an advocate of game theory until the early 2000's.

They have co-authored two game theory textbooks.

Game Theoretic Modeling Process

Real World

Game Theoretic Modeling Process

Real World **Math World**

Game Theoretic Modeling Process

Real World

Phenomenon (a scenario involving interacting agents)

Math World

Game Theoretic Modeling Process

Real World

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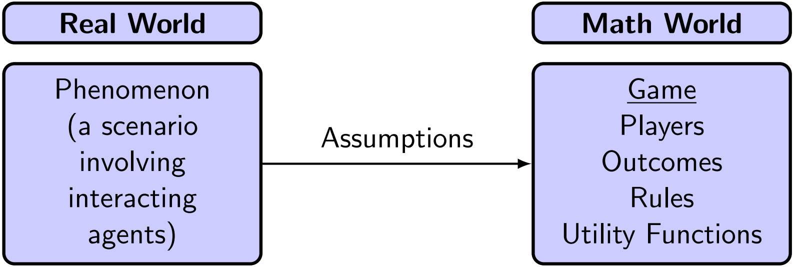
Math World

Game
Players
Outcomes
Rules
Utility Functions

Game Theoretic Modeling Process



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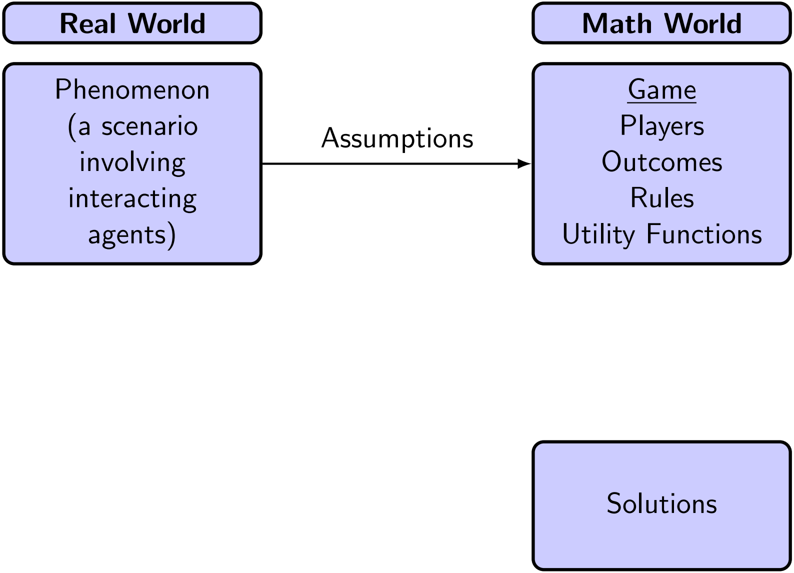

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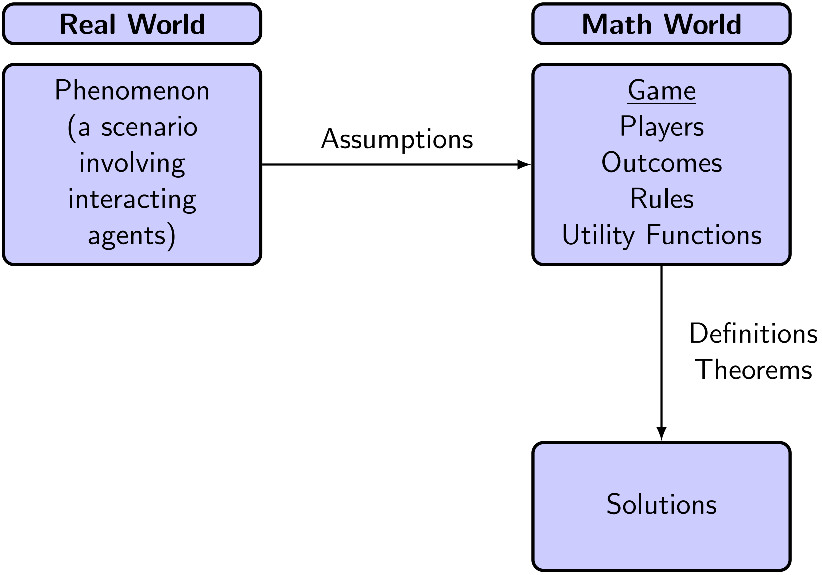

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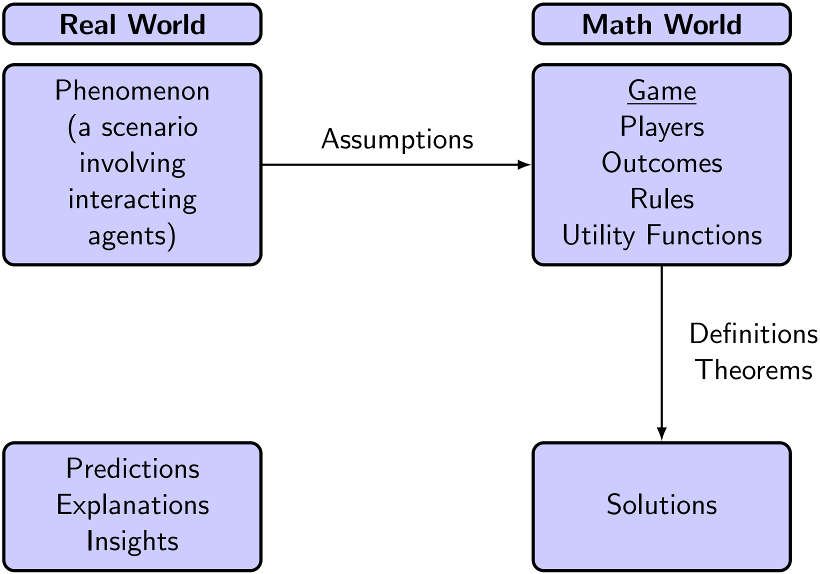

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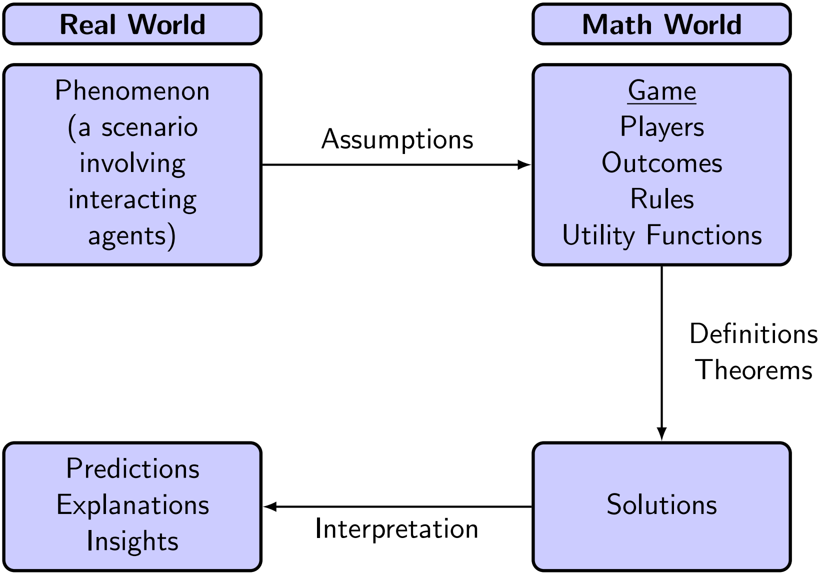

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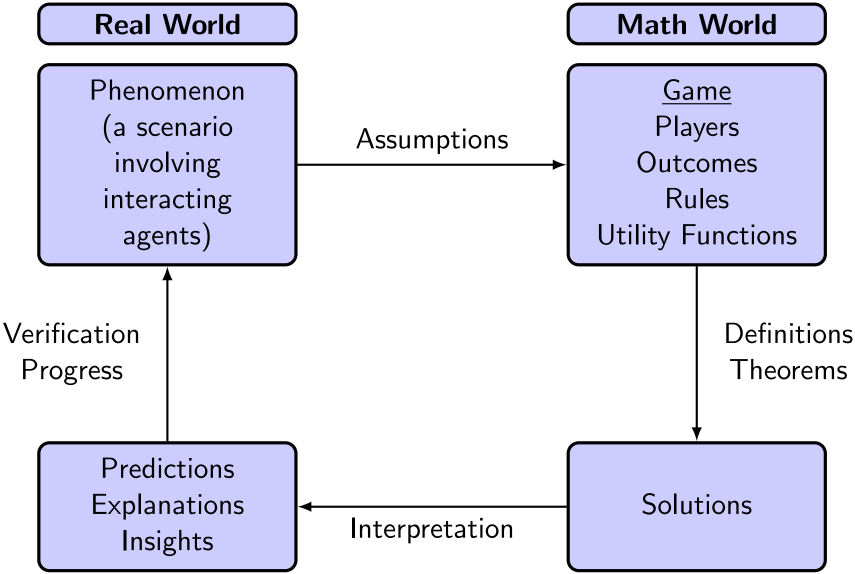

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Abbreviation	Outcome
$(10, 0)$	You win \$10 and opponent wins \$0
$(6, 6)$	You win \$6 and opponent wins \$6
$(2, 2)$	You win \$2 and opponent wins \$2
$(0, 10)$	You win \$0 and opponent wins \$10

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- Only one randomly chosen pair will play for real money.

First Scenario Utilities

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$(6, 6)$	You win \$6 and opponent wins \$6
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Question	Outcome.	Utility
If you were offered the four possible outcomes, which one would you choose?		
If you were offered the three remaining outcomes, which one would you choose?		
If you were offered the two remaining outcomes, which one would you choose?		
What is the remaining outcome?		

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Outcome		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

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Utility		Colin A	Colin B
Rose	A		
	B		

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Utility		Colin A	Colin B
Rose	A		
	B		

4. In private, write "A" or "B" on your index card.

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3. Outcome and utility tables:

Outcome		Colin A	Colin B
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	B	0, 10	6, 6

Utility		Colin A	Colin B
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5. Show your choices to reveal the outcome of the game.

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6. Discuss choices made by players.

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Outcome		Colin A	Colin B
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Utility		Colin A	Colin B
Rose	*A*		
	B		

4. In private, write "A" or "B" on your index card.
5. Show your choices to reveal the outcome of the game.
6. Discuss choices made by players.
7. Participation points as an alternative to money in actual classes.

Strategic Game Analysis I

Outcomes		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Self A	Colin Self B
Rose	A	20, 20	100, 0
Self	B	0, 100	60, 60

- Prudential strategies: A for Rose and A for Colin
- Dominant Strategies: A for Rose and A for Colin
- Nash equilibrium: (A, A)
- Efficient: (B, B), (A, B), and (B, A)
- Prisoner's Dilemma Game

Strategic Game Analysis II

Outcomes		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Equal A	Colin Equal B
Rose Equal	A	4, 4	0, 0
	B	0, 0	10, 10

- Prudential strategies: both for Rose and both for Colin
- Dominant Strategies: neither for Rose and neither for Colin
- Nash equilibrium: (A, A), (B, B), and $(\frac{5}{7}A + \frac{2}{7}B, \frac{5}{7}A + \frac{2}{7}B)$
- Efficient: (B, B)
- Coordination Game

Strategic Game Analysis III

Outcomes		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Equal A	Colin Equal B
Rose	A	0, 4	10, 0
Mixed	B	3, 0	8, 10

- Prudential strategies: B for Rose and both for Colin
- Dominant Strategies: neither for Rose and neither for Colin
- Nash equilibrium: $(\frac{5}{7}A + \frac{2}{7}B, \frac{2}{5}A + \frac{3}{5}B)$
- Efficient: (A, B) and (B, B)

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3. If you do not have a signed agreement within the available time, player A gets $4, player Z gets nothing, and Rick keeps the rest.

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4. Only one pair will be playing for real.
5. Assume players are money and candy loving, goods separable, and risk neutral. Then $u(\text{money}) > 0$, $u(\text{candy}) > 0$, and $u(a \cdot \text{money} + b \cdot \text{candy}) = au(\text{money}) + bu(\text{candy})$ for $0 \leq a, b \leq 1$.

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6. So, for each player we only need to determine $u(\text{money})$ and $u(\text{candy})$ and can choose them to sum to 100.

Issue	A's Utility	Z's Utility
Money		
Candy		
Total	100	100

Second Scenario Special Feasible Utility Pairs

Issue	A's Utility	Z's Utility
Money	40	10
Candy	60	90
Total	100	100

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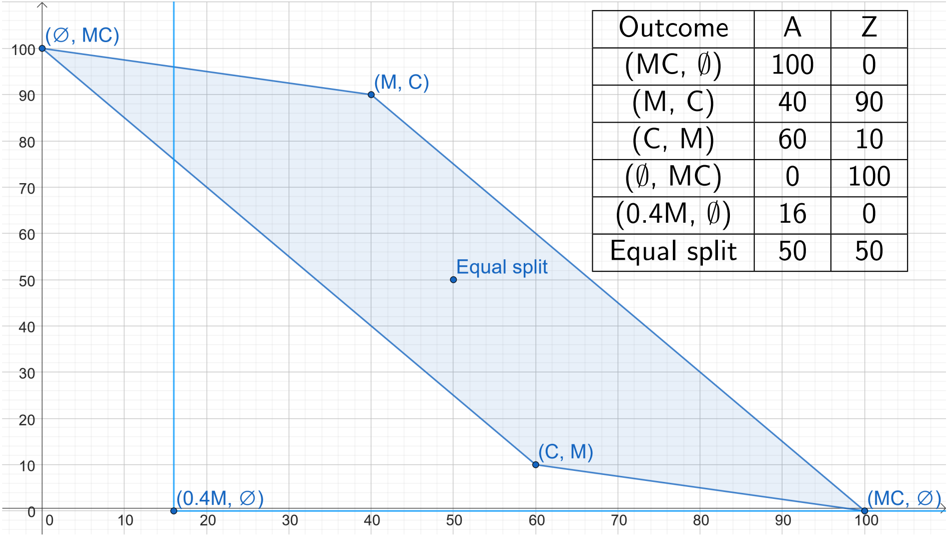
Total	100	100

Outcome Name	Issue Winner Money	Issue Winner Candy	A's Utility	Z's Utility
(MC, ∅)	A	A	100	0
(M, C)	A	Z	40	90
(C, M)	Z	A	60	10
(∅, MC)	Z	Z	0	100
(0.4M, ∅)	0.4A	-	16	0
Equal Split	tie	tie	50	50

Second Scenario Feasible and Rational Utility Pairs



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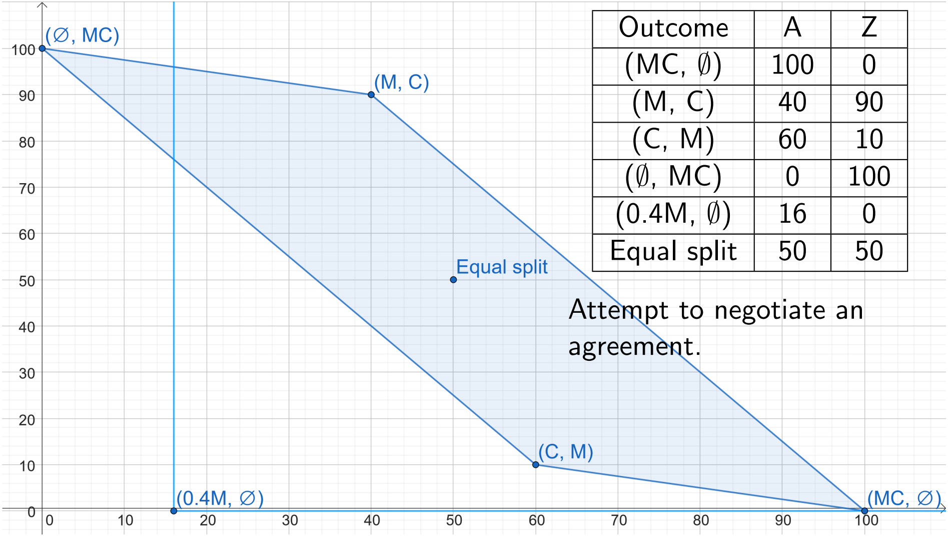
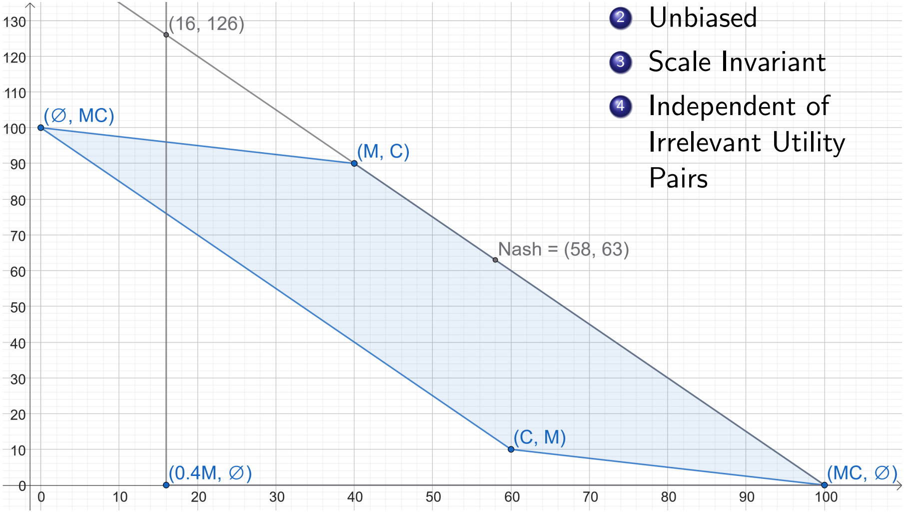


Outcome	A	Z
(MC, ∅)	100	0
(M, C)	40	90
(C, M)	60	10
(∅, MC)	0	100
(0.4M, ∅)	16	0
Equal split	50	50

Attempt to negotiate an agreement.

Nash Bargaining Game Solution



1. Efficient
2. Unbiased
3. Scale Invariant
4. Independent of Irrelevant Utility Pairs

Conclusions

1. You have experienced two activities that introduce strategic and bargaining games.
2. In most textbooks, most concepts are introduced with a story/scenario, so you only need to build the activity.
3. There is a time trade-off between the number of activities and the number of topics covered.