

Game Theoretic Modeling for Math Majors

Strategic Games

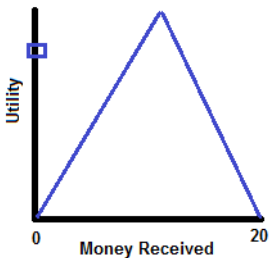
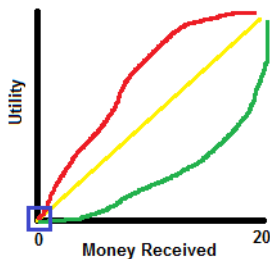
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Utility Functions

- A randomly selected pair of participants will have the opportunity to split \$20 or both to receive nothing.
- On an index card, write your name on one side and your vNM utility function for the possible outcomes on the other side:
 1. $\$x$ given to you and $\$(20 - x)$ given to the other person (the graph), or
 2. nothing given to both (the square).
- The cards will be collected and the vNM utilities (but not names) will be announced publicly.
- In the discussion that follows, you do not need to reveal which vNM utilities are yours.



Ultimatum Game

- Obtain two volunteers (proposer and responder) to play for money, and publicly reveal their utility functions.
- The proposer offers x of \$20 to the responder.
- The responder either accepts or rejects the offer.
- If the offer is accepted, the responder receives x and the proposer receives $20 - x$.
- If the offer is rejected, the proposer and responder receive nothing.
- Play the game.
- Think: Observations? Questions?
- Pair: Discuss with one or two others.
- Share: Report summaries/details.
- Complete information game and its Nash equilibria.
- Experimental observations.

Ultimatum Game Again

- One more pair of participants will play the Ultimatum Game against a randomly chosen other willing participant, but (if there is enough time) we will also see what would happen to every other pair who plays.
- Everyone willing to play this game should write on an index card:
 - Their name.
 - I offer ____ of the \$20 to the responder.
 - I will accept any offer that satisfies ____.
- Collect the cards, randomly pair them, and announce the statements. Only the first pair is playing the game for actual cash.
- Discuss.

Incomplete Information Game Model

Assumptions:

- The proposer can offer 1, 2, or 3 of 4 points.
- q self-interested risk-neutral player types (SI).
- $1 - q$ equity-seeking player types (ES).
- Each player knows their own type but not the type of the other player.

Outcome (me, you)	Utilities	
	SI	ES
(3, 1)	3	1
(2, 2)	2	3
(1, 3)	1	0
(0, 0)	0	2

Actions and Payoffs

Prop. Offer	Resp. SI	Action ES	Proposer Payoff SI Type	ES Type
1	accept	reject	$3q + 0(1 - q) = 3q$	$1q + 2(1 - q) = 2 - q$
2	accept	accept	$2q + 2(1 - q) = 2$	$3q + 3(1 - q) = 3$
3	accept	reject	$1q + 0(1 - q) = q$	$0q + 2(1 - q) = 2 - 2q$

- If $q < 2/3$, even self-interested players should offer 2 points.
- If $q > 2/3$, self-interested players should offer 1 point.

- Utility function construction is a continuing theme
- Strategic games with complete and incomplete information
- Active student involvement
- Balance making activities personal without forcing self-revelation
- Index cards for recording and randomization
- Class participation points rather than money requires everyone to play
- Discussion protocols