Preferences in Game Theory

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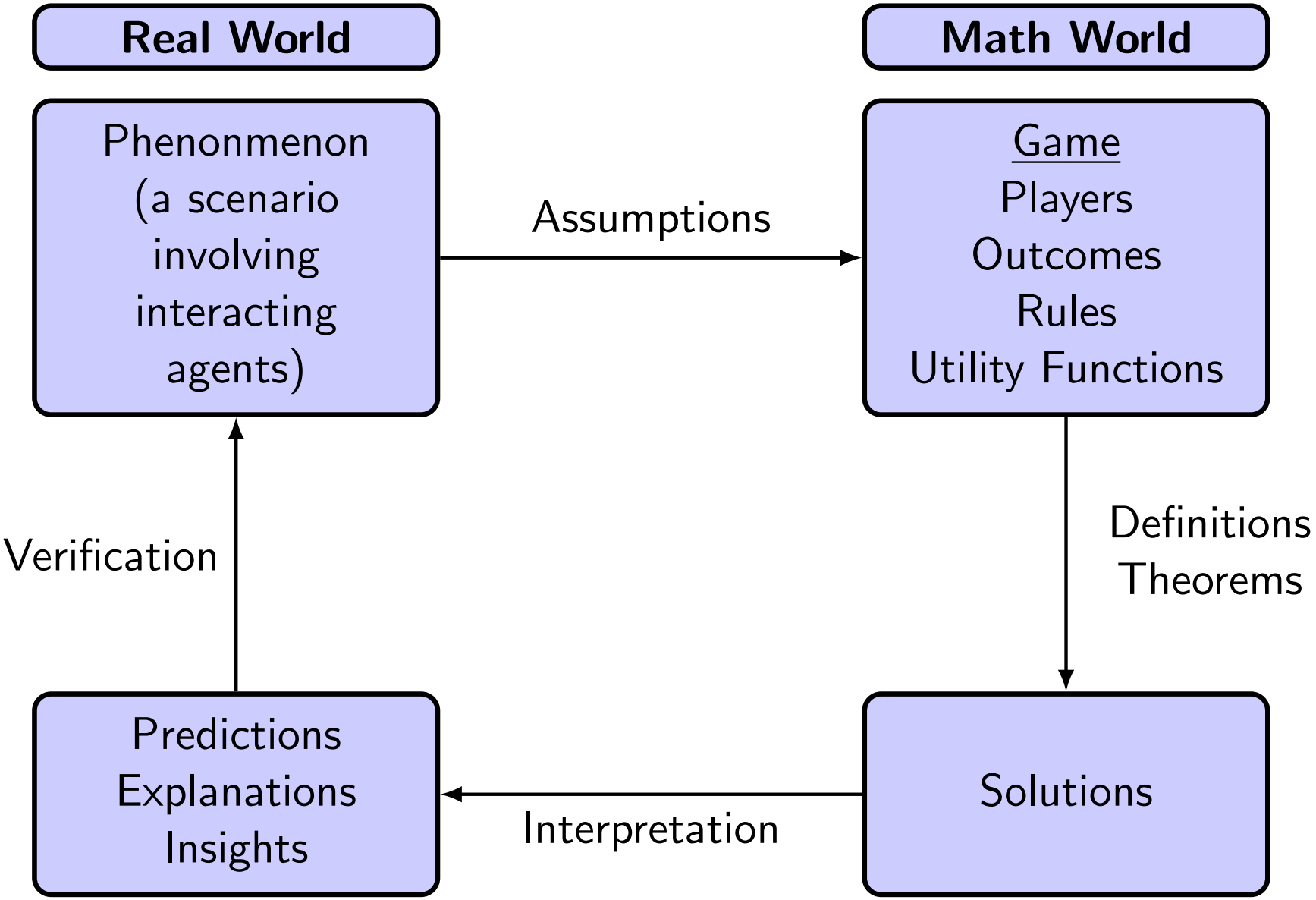
MAA Indiana Section Fall 2023 Meeting

These slides can be downloaded from www.goshen.edu/dhousman under the Presentations section

Abstract

Game theory is the mathematical modeling of scenarios involving two or more decision makers, and it has been used successfully in economics, political science, biology, and other disciplines. Nonetheless, some game theory models have also been criticised for failing to accurately predict actual real-world behavior. The goal of this talk is to suggest that game theory models and solutions may not be at fault. Instead failures may be due to unwarranted assumptions about decision maker preferences over possible outcomes. This will be illustrated with a simple scenario in which the outcomes involve audience members receiving money, the amount depending on their choices.

Game Theoretic Modeling Process



Our Scenario Outcomes

Abbreviation	Outcome
$(10, 0)$	You win \$10 and another wins \$0
$(6, 6)$	You win \$6 and another wins \$6
$(2, 2)$	You win \$2 and another wins \$2
$(0, 10)$	You win \$0 and another wins \$10

- If you were offered these four possible outcomes, which one would you choose? Write this outcome on the top line of your index card.
- If you were offered the three remaining outcomes, which one would you choose? Write this outcome on the second line of your index card.
- If you were offered the two remaining outcomes, which one would you choose? Write this outcome on the third line of your index card.
- Write the remaining outcome on the fourth line of your index card.
- Write to the right of your list of outcomes the numbers 4, 3, 2, 1. These numbers are your ordinal utilities we will use to model your unconstrained choices among subsets of these outcomes.

Examples of Ordinal Utilities

Rank	1	2	3	4
Ordinal Utility	4	3	2	1
Self Interested	$(10, 0)$	$(6, 6)$	$(2, 2)$	$(0, 10)$
Other Interested	$(0, 10)$	$(6, 6)$	$(2, 2)$	$(10, 0)$
Vindictive	$(10, 0)$	$(2, 2)$	$(6, 6)$	$(0, 10)$
Group then Self	$(6, 6)$	$(10, 0)$	$(0, 10)$	$(2, 2)$
Group then Other	$(6, 6)$	$(0, 10)$	$(10, 0)$	$(2, 2)$
Equal then Self	$(6, 6)$	$(2, 2)$	$(10, 0)$	$(0, 10)$
Equal then Other	$(6, 6)$	$(2, 2)$	$(0, 10)$	$(10, 0)$
$v(x, y) = 2x + y - \lvert x - y \rvert$	$(6, 6)$	$(10, 0)$	$(2, 2)$	$(0, 10)$

The Rules

1. No communication, coercion, or binding agreements.
2. Write "A" or "B" on your index card (but do not do it yet).
3. Two people will be chosen at random.
4. They will reveal their choices, and money will be distributed in the following manner:
   - If both choose A, then both receive $2.
   - If both choose B, then both receive $6.
   - If they make different choices, then A receives $10 and B receives $0.
5. Make your choice now!

Strategic Games and Nash Equilibria

Outcomes	A	B
A	(2, 2)	(10, 0)
B	(0, 10)	(6, 6)

(Self, Self)	A	B
A	(2, 2)	(4, 1)
B	(1, 4)	(3, 3)

(Self, Self)	A	B
A	($\boxed{2}$, $\boxed{2}$)	($\boxed{4}$, 1)
B	(1, $\boxed{4}$)	(3, 3)

(Self, v)	A	B
A	($\boxed{2}$, $\boxed{2}$)	($\boxed{4}$, 1)
B	(1, 3)	(3, $\boxed{4}$)

(v, v)	A	B
A	($\boxed{2}$, $\boxed{2}$)	(3, 1)
B	(1, 3)	($\boxed{4}$, $\boxed{4}$)

But in our scenario, the players only knew their opponent probabilistically, which is better modeled as a game with incomplete information.

Two Relevant Texts