Game Theory: Activities Motivate Concepts

MathFest Workshop

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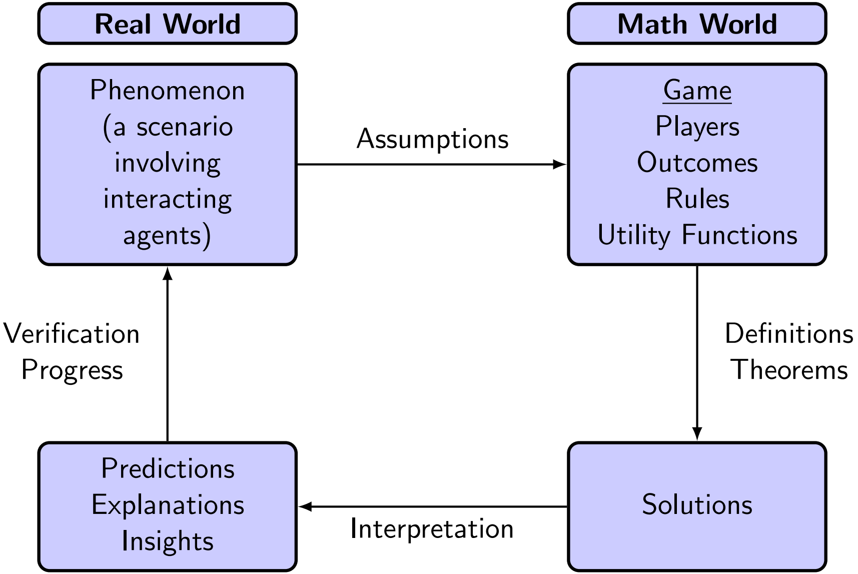
Presenters

David is a professor of mathematics and computer science at Goshen College in northern Indiana. His Ph.D. is in game theory. He has taught modeling and game theory courses throughout his career.

Rick is a retired professor of mathematics from Valparaiso University (in northwest Indiana) and he did not become an advocate of game theory until the early 2000's.

They have co-authored two game theory textbooks.

Game Theoretic Modeling Process



First Scenario Players and Outcomes

- As much as possible, the scenario will be described as a game: players, rules, outcomes, and utilities.
- You will be one of two players. Pair up now!
- Here are the possible outcomes:

Abbreviation	Outcome
$(10, 0)$	You win \$10 and opponent wins \$0
$(6, 6)$	You win \$6 and opponent wins \$6
$(2, 2)$	You win \$2 and opponent wins \$2
$(0, 10)$	You win \$0 and opponent wins \$10

- Only one randomly chosen pair will play for real money.

First Scenario Utilities

Abbreviation	Outcome
$(10, 0)$	You win \$10 and opponent wins \$0
$(6, 6)$	You win \$6 and opponent wins \$6
$(2, 2)$	You win \$2 and opponent wins \$2
$(0, 10)$	You win \$0 and opponent wins \$10

Question	Outcome	Utility
If you were offered the four possible outcomes, which one would you choose?		
If you were offered the three remaining outcomes, which one would you choose?		
If you were offered the two remaining outcomes, which one would you choose?		
What is the remaining outcome?		

First Scenario Utilities Reporting

Question	Outcome	Utility
Choice among all four?	(6, 6)	100
Choice among three remaining?	(2, 2)	80
Choice among two remaining?	(10, 0)	15
What is the remaining outcome?	(0,10)	0

Question	Answer
Email	equity@college.edu
Name	Equity
(10, 0)	15
(6, 6)	100
(2, 2)	80
(0, 10)	0

Submit your utilities

`http://goshen.edu/game-theory`

First Scenario Rules

1. A pair of volunteers will play the game for real in front of everyone.
2. Each player will privately write "A" or "B" on their handout.
3. No coercion or binding agreements are allowed.
4. Each player in the pair should look at each other's utilities. The utility or payoff matrix summarizes the strategic game model.

Outcome		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Utility		Colin A	Colin B
Rose	A		
	B		

5. After looking at each other's utilities, privately write "A" or "B" on your handout.
6. Finally, show each other your choices to reveal the outcome.
7. Discuss choices made by players.
8. Participation points as an alternative to money in actual classes.

Strategic Game Analysis I

Outcomes		Colin A	B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Self A	B
Rose	A	20, 20	100, 0
Self	B	0, 100	60, 60

- Both players self-interested and risk neutral.
- Prisoner's Dilemma Game
- Prudential strategies: A for Rose and A for Colin
- Dominant Strategies: A for Rose and A for Colin
- Nash equilibrium: (A, A)
- Efficient: (B, B), (A, B), and (B, A)

Strategic Game Analysis II

Outcomes		Colin A	B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Equal A	B
Rose Equal	A	40, 40	0, 0
	B	0, 0	100, 100

- Both players want equality and are slightly risk adverse.
- Coordination Game
- Prudential strategies: both for Rose and both for Colin
- Dominant Strategies: neither for Rose and neither for Colin
- Nash equilibrium: (A, A), (B, B), and $(\frac{5}{7}A + \frac{2}{7}B, \frac{5}{7}A + \frac{2}{7}B)$
- Efficient: (B, B)

Strategic Game Analysis III

Outcomes		Colin A	Colin B
Rose	A	2, 2	10, 0
	B	0, 10	6, 6

Payoffs		Colin Equal A	Colin Equal B
Rose	A	0, 40	100, 0
Mixed	B	30, 0	80, 100

- Rose predominantly values her money but also values Colin's money.
- Prudential strategies: B for Rose and both for Colin
- Dominant Strategies: neither for Rose and neither for Colin
- Nash equilibrium: $(\frac{5}{7}A + \frac{2}{7}B, \frac{2}{5}A + \frac{3}{5}B)$
- Efficient: (A, B) and (B, B)

Second Scenario

1. Same as the first scenario except that you will be matched randomly with someone else in this room.
2. Outcomes remain:

Outcome		Colin	
		A	*B*
Rose	*A*	2, 2	10, 0
	B	0, 10	6, 6

3. Each person will submit an index card with (a) the name or pseudonym they used earlier, and (b) their strategy choice.
4. We will shuffle the cards and have pairs of cards play against each other.
5. Only the last pair will be playing for real.
6. The uncertainty players have about the utilities of their opponent makes this a Bayesian or incomplete information game.
7. Before you make your choice, we will provide you with information about everyone's utilities.

Bayesian Equilibrium

1. Suppose $c = (c_1, c_2, \ldots, c_n)$ are the choices by the n players.
2. Suppose a_i and b_i are the number of A and B choices by the players other than i.
3. A is a best response by player i to c if $a_i u_i(AA) + b_i u_i(AB) \geq a_i u_i(BA) + b_i u_i(BB)$.
4. B is a best response by player i to c if $a_i u_i(AA) + b_i u_i(AB) \leq a_i u_i(BA) + b_i u_i(BB)$.
5. $c = (c_1, c_2, \ldots, c_n)$ is a Bayesian equilibrium if c_i is a best response by player i to c for all players i.

Conclusions

1. You have experienced two activities that introduce strategic and Bayesian games.
2. In most textbooks, most concepts are introduced with a story/scenario, so you only need to build the activity.
3. There is a time trade-off between the number of activities and the number of topics covered.
4. Outcomes could involve physical prizes or points towards the course grade.
5. Consider the level of anonymity for participants.
6. These slides and information about the second scenario game results will be emailed within a week.