A Mathematical Modeling Course

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Course Overview

- Project-centered course.
- Topics: functional relationship, optimization, and randomness.
- Prerequisite: One upper-level course in Linear Algebra, Differential Equations, or Probability and Statistics. One programming course.
- For each topic, seven 75-minute classes devoted to relevant mathematics, one class for project consultation, and one class for project presentations.
- Grade is based on three projects (60%) and many small assignments (40%).

First Class: Modeling

- What is the maximum number of tennis balls that will fit in this room?
- Individual responses after 1 minute range over 3-5 orders of magnitude.
- Group responses after 20 minutes with measuring instruments and tennis balls are usually all within one order of magnitude.
- Learning objectives: measurement precision, making assumptions explicit, and the possibility of multiple correct answers.
- Repeat as an individual homework assignment due at the start of next class. One hour time limit.

Second Class: Assessment

- Written reports are often used to communicate the answer to a question. In this course, such reports will include a formulation, analysis, and interpretation of a mathematical model. By learning how to assess the quality of such reports, you will be able to assess and improve the quality of your own project reports.
- Groups of students review with guided critical thinking questions ...
  - Information about a mathematical modeling report.
  - Information about assessment.
  - An example of a tennis ball report.
  - An example of an assessment of the report.
- As a skill development exercise, each group provides assessments of other student reports handed in at the start of class.

Functional Relationships

- Study a phenomenon that involves a relationship among three or more variables. Build a mathematical model of the relationship based upon reasonable assumptions about the phenomenon. Gather data in order to specify and verify the model. Present your work in a written report and a class presentation.
- Examples:
  - The time that bubbles survive as a function of humidity, temperature, ratio of soap to water, and/or bubble diameter.
  - Time for an object to fall to the ground as a function of initial height, weight, cross sectional area, density, and/or shape.
  - The weight of earthworms as a function of length and maximum circumference.
- Mathematics: proportionality assumptions to derive equations, error propagation, fitting model parameters to data via regression, software (Mathematica).

Optimization

- Study a real world problem involving optimization. Describe an appropriate mathematical model based upon reasonable assumptions. Obtain a solution based upon the model. Gather data in order to test your solution. Test and critique your solution. Present your work in a written report and a class presentation.
- Examples:
  - Choose a schedule of retrievals of wounded persons that maximizes the number of persons saved.
  - Determine the best pairing of students to double dorm rooms given their personal attributes on a Likert scale.
  - Choose a flight path from an initial airport to a final airport that minimizes distance traveled subject to safety and fuel constraints.
- Grading weights depend upon the emphasis taken in the project.
- Mathematics: Calculus, Lagrange multipliers, greedy and brute force algorithms, MCM problem, software (Mathematica).

Randomness

- Study a real world problem involving randomness. Describe an appropriate mathematical/simulation model based upon reasonable assumptions. Obtain results based upon simulation runs and/or mathematical analysis of your model. Summarize and critique your results. Present your work in a written report and a class presentation.
- Mathematics: random variables, simulation software (NetLogo), game theory (incomplete information).
- Two examples that were reported during the Goshen College Academic Symposium:
  - Spencer Aeschliman, Luke Rush and Jordan Haarer, Forest Fire Simulation
  - Kartikeya Sharma, Confirming Tolerance: A Study of Tolerance of American States with Schellings Model of Segregation

Questions?

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