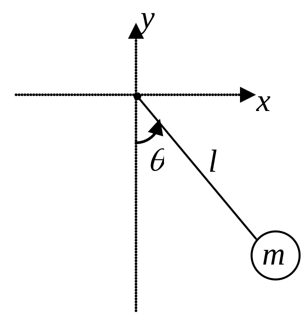
Pendulum



Assumptions

1. $\vec{F} = m\vec{a}$ (Newton's second law of motion)
2. $\vec{F} = \vec{F}_{\text{gravity}} + \vec{F}_{\text{rod}} + \vec{F}_{\text{drag}} + \vec{F}_{\text{external}}$ (specifying forces)
3. $\vec{F}_{\text{gravity}} = mg < 0, -1 >$ (constant downward pull)
4. $\vec{F}_{\text{rod}}$ is such that the distance between pivot and bob is a constant
5. $\vec{F}_{\text{drag}} = -b\vec{v}$
6. $\vec{F}_{\text{external}} = F\cos(\omega t) < 0, 1 >$ (action on the pivot)

Consider the <*x*, *y*> position of the bob as a function of *θ*.

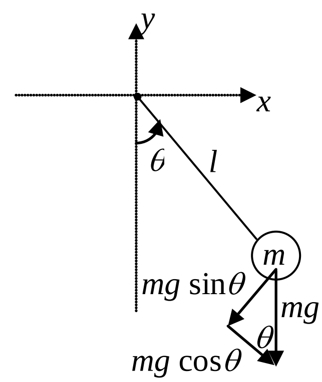
$$\text{position} = \vec{p} = l < \sin\theta, -\cos\theta >$$

$$\text{velocity} = \vec{v} = l\dot{\theta} < \cos\theta, \sin\theta >$$

$$\text{acceleration} = \vec{a} = l\ddot{\theta} < \cos\theta, \sin\theta > + l(\dot{\theta})^2 < -\sin\theta, \cos\theta >$$

By assumptions 1 & 2:

$$m\vec{a} = \vec{F}_{\text{gravity}} + \vec{F}_{\text{rod}} + \vec{F}_{\text{drag}} + \vec{F}_{\text{external}}$$



In the direction of motion,

$$ml\ddot{\theta} = -mg\sin\theta + 0 - bl\dot{\theta} + F\cos(\omega t)\sin\theta$$

As a system,

$$\dot{\theta} = v$$

$$\dot{v} = -(g/l)\sin\theta - (b/m)v + (F/ml)\cos(\omega t)\sin\theta$$

Choose units so that $g = l = m = 1$:

$$\dot{\theta} = v$$

$$\dot{v} = -\sin\theta - bv + F\cos(\omega t)\sin\theta$$

Compare with a forced spring-mass system:

$$\dot{x} = v$$

$$\dot{v} = -x - bv + F\cos(\omega t)$$