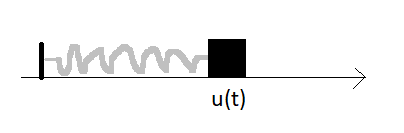
**Math 321 Differential Equations**

**Spring-Mass System**



Suppose is the displacement of the mass from its resting position at time .

Assumptions

1. (Newton’s Second Law of Motion) Acceleration of the mass is proportional to the applied force, and the reciprocal of the proportionality constant is what we call mass.

1. The forces being applied to the mass are the spring, drag due to friction, and an externally applied force. Forces are additive.
2. (Hooke’s Law) The spring force is proportional to the displacement of the mass from the resting position of the spring and is directed toward the resting position.
3. The drag force is proportional to the velocity of the mass and is directed opposite of the direction of motion.
4. The external force is given by some function of time.

Derivation

Combining assumptions, we obtain

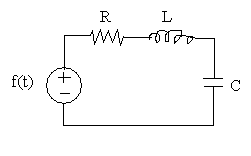
which can be written

This is a second order linear with constant coefficients ordinary differential equation. If , then the equation is homogeneous; otherwise, it is non-homogeneous.

If the spring-mass system is turned to be vertical, the spring attachment point is moved in a time dependent manner (say by a motor), is the downward displacement of the mass from its resting position at time , and is the downward displacement of the spring attachment point from its resting position at time , then the resulting differential equation is

**Math 321 Differential Equations**

**RLC Circuit**



Let be the charge accumulated at the capacitor.

Assumptions

1. (Kirchhoff’s Voltage Law) The sum of voltage changes around a closed loop must be zero.
2. (Ohm’s Law) The voltage drop across a resistor is proportional to the current flowing through the resistor. Current is the instantaneous change in charge accumulation.
3. The accumulation of charge at a capacitor is proportional to the voltage across it.
4. The voltage across an inductor is proportional to the rate of change of the current.
5. The voltage supplied by the source is some function of time.

Derivation

Combining assumptions, we obtain

which can be rewritten

and can be compared with the spring-mass equation

Note also that the Hill-Keller differential equation can be written

where is the position of the runner at time .